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CONDENSATION OF VAPOUR BUBBLES IN LIQUID

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Аннотация—На основании первого закона термодинамики переменной массы исследуется конденсация сферически симметричного парового пузыря, находящегося в неподвижной недогретой жидкости. Полученные теоретические результаты сравниваются с имеющимися экспериментальными данными разлщцных авторов.

NOMENCLATURE

k, adiabatic exponent of vapours;
i, enthalpy;
G, mass change/s;
τ, time;
p_s, pressure of saturated vapour;
R, radius;
σ, surface tension;
λ, thermal conductivity;
a, thermal diffusivity;
ΔT, degree of superheating, $= T_s - T_l$
ρ, density;
r, heat of evaporation;
c, heat capacity;
V, volume;
Q, heat quantity;
$\overline{R}$, (R/R_0);
T, t, temperature [°K, °C].

Subscripts
$'$, refers to liquid;
$''$, to vapour;
0, initial value of parameter;
i, input;
th, output;
l, liquid.

CONDENSATION of spherical vapour bubbles has been studied many times both theoretically and experimentally [1–10]. Theoretical work for the calculation of the rate of collapse of vapour bubbles is based either on the solution of the equation of motion of the bubble wall in the Rayleigh hydrodynamic model with the use of the Plesset–Zwick temperature integral [8], or on the solution of the equation for the Bosnjakovic analytical model. Nevertheless, the solutions obtained which, as a rule, are very complicated or most often numerical, are in satisfactory agreement with the experimental results of the authors of [1, 5, 9] or with the results of [3].

Up till now no comprehensive comparison of the theoretical results with experimental data of the various investigations has been carried out. Recent experiments on collapse of vapour bubbles in liquid nitrogen [6], as we shall see later on, are in a poor agreement with theoretical equations. In the present work condensation of bubbles in subcooled liquid is considered theoretically on the basis of the first law of thermodynamics for a system with variable mass which was first successfully used by the authors of the present paper for the calculation of vapour bubble growth [12].

Let us consider a bubble in subcooled liquid. With the assumption that vapour obeys the gas laws, the equation of unsteady thermodynamics may be written as follows [14]

$$\frac{dp}{d\tau} = \frac{k-1}{V} \times \left(\frac{dQ}{d\tau} + i_i G_i - i_{th} G_{th} - \frac{k}{k-1} p \frac{dV''}{d\tau}\right). \quad (1)$$

Vapour pressure p is determined by the known

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formula from [13]

$$p = p_s + (2\sigma/R). \quad (2)$$

Since the temperature of vapour in a bubble is higher than the temperature of the surrounding liquid, the heat necessary for condensation of an elementary volume dV is removed from the bubble. This heat may be calculated from the equation [10]

$$\frac{dQ}{d\tau} = -\frac{\lambda'\Delta T}{\sqrt{(\pi a'\tau)}} 4\pi R^2. \quad (3)$$

The value of energy input $i_i G_i = 0$ since in condensation the reverse process of liquid evaporation may be neglected.

The amount of energy leaving the bubble volume together with the condensed vapour is [the heat liberated in condensation is accounted for by equation (3)]

$$i_{th} G_{th} = i'\rho'' \frac{dV}{d\tau}. \quad (4)$$

Here i' is the enthalpy of liquid at temperature T_s calculated from the temperature T_l. The elementary volume dV'' is equal to the change in bubble volume dV minus the volume occupied by the condensed vapour, i.e.

$$dV'' = dV - dV' = [1 - (\rho''/\rho')]\, dV. \quad (5)$$

The adiabatic vapour exponent is determined from equation [15]

$$k = \left(1 - 2\frac{p_s}{r\rho''}\right)^{-1}. \quad (6)$$

Using the relationships (2)–(6) we may transform the initial equation (1) to the form

$$\left(\frac{1}{\varepsilon} + \frac{\varkappa}{R}\right)\frac{dR}{d\tau} = -\frac{2}{\sqrt{\pi}} Ja\sqrt{(a'/\tau)}. \quad (7)$$

Here

$$\frac{1}{\varepsilon} = f_\rho + \frac{2i'h}{r} \quad \varkappa = \frac{2\sigma}{p_s}\left(f_\rho - \frac{1}{3} + \frac{2}{3}N_1\right);$$

$$N_1 = \frac{p_s}{r\rho''}; \quad Ja = \frac{\Delta T c'\rho'}{r\rho''}; \quad f_\rho = 1 - \frac{\rho''}{\rho'}.$$

If $f_\rho \cong 1$ (which holds almost always at pressures far from the critical pressure), the expression for $\varkappa$ is simplified

$$\varkappa = \frac{4}{3}(\sigma/p_s)(1 + N_1).$$

Table 1.

t (°C)	0	50	100	150	200
$\varkappa$ (M)	$1{\cdot}653.10^{-4}$	$7{\cdot}35.10^{-6}$	$7\,73.10^{-7}$	$1{\cdot}345.10^{-7}$	$3{\cdot}565.10^{8}$
N_1	0·0502	0·06212	0·0751	0·0884	0·1019
A (m²/s. deg²)	-	$8{\cdot}68.10^{-5}$	$1{\cdot}935.10^{-6}$	$1{\cdot}156.10^{-7}$	$1{\cdot}41.10^{-8}$

For water the values of N_1 and $\varkappa$ are presented in Table 1. On integration of (7) from 0 to τ and from R_0 to R

$$\bar{R} + \varkappa\varepsilon \ln \bar{R} = 1 - \frac{4}{\sqrt{\pi}} \varepsilon Ja \frac{\sqrt{(a'\tau)}}{R_0}. \quad (8)$$

On introduction of the dimensionless time as follows [5]

$$\bar{\tau} = \frac{4}{\pi} Ja^2 \frac{a'\tau}{R_0^2} = A\left(\frac{\Delta T}{R_0}\right)^2 \tau. \quad (9)$$

The group containing physical parameters of liquid and vapour

$$A = \frac{4}{\pi} a' \left(\frac{c'\rho'}{r\rho''}\right)^2$$

depends on temperature and for water is also given in Table 1. Then (8) will be of the form

$$\bar{R} + \varkappa\varepsilon\bar{R} = 1 - 2\varepsilon\sqrt{\bar{\tau}}. \qquad (10)$$

At $(2\sigma/R) \ll p_s$ this formula is simplified

$$\bar{R} = 1 - 2\varepsilon\sqrt{\bar{\tau}}. \qquad (11)$$

For comparison of the calculations obtained by the above equations with experiments, experimental results of a number of authors on the collapse of vapour bubbles have been employed. In Figs. 1 and 2 experimental points of Akiyama

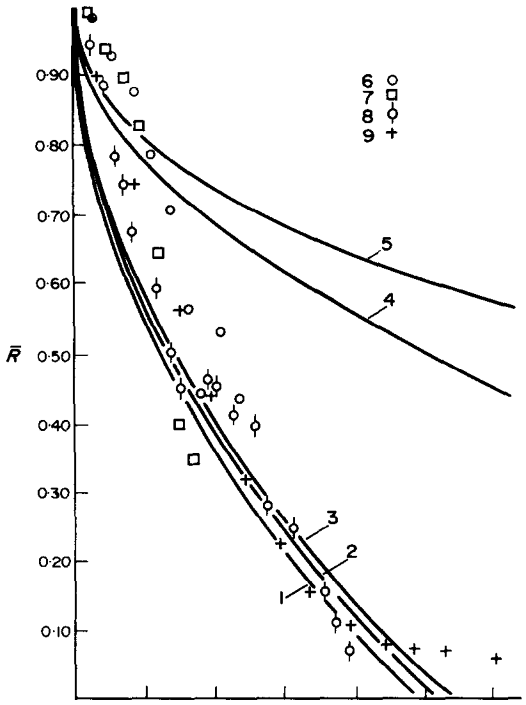

FIG. 1. Comparison of theoretical and experimental results on bubble condensation in water:
1. Calculation by formula (11) at $\varepsilon = 1$ ($\Delta T = 0$);
2. Calculation by formula (11) at $\varepsilon = 0{\cdot}978$ ($\Delta T = 11°$);
3. Calculation by formula (11) at $\varepsilon = 0{\cdot}96$ ($\Delta T = 22°$);
4. Calculation by formula (12);
5. Calculation by (13).

○□ experimental points of Akiyama [1] at $t_s = 100°C$ and 102°C and at $\Delta T = 21{\cdot}5°$ and $\Delta T = 22°$, respectively.

ϕ experimental points of Harrach [1] at $t_s = 100°C$ and $\Delta T = 11°$.

+ experimental points of Levenspiel [5].

[1], Harrach [1], Levenspiel [5] obtained for condensation of vapour bubbles in water and by Hewitt–Parker [8] for condensation of vapour

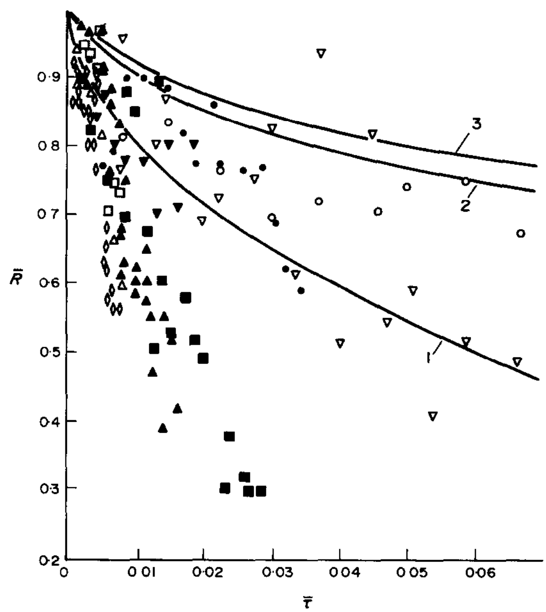

FIG. 2. Comparison of experimental data on bubble collapse in liquid nitrogen [6] with equations (11) curve I, (12) curve 2, (13) curve 3 at $\Delta T = 2{\cdot}06° + 0{\cdot}41°$.

bubbles in liquid nitrogen are presented. The curves I calculated by (11) at $\varepsilon = \mathrm{I}(\Delta T = 0)$; 2, by (11) at $\varepsilon = 0{\cdot}978$ ($\Delta T = 11°$); 3, by (11) at $\varepsilon = 0{\cdot}96$ ($\Delta T = 22$); 4 and 5, by equations of Florschuetz–Chao [5]

$$\bar{R} = 1 - \sqrt{\bar{\tau}} \qquad (12)$$

$$\bar{\tau} = \frac{1}{3}[(2/\bar{R}) + \bar{R}^2 - 3] \qquad (13)$$

are also plotted in these figures.

It is easy to see that the equations presented here give better agreement with experiments than the available formulae.

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Abstract—Based on the first law of thermodynamics for system with variable mass, condensation of a spherically symmetric vapour bubble in fixed sub-cooled liquid is studied. The theoretical results obtained are compared with the available experimental data of various authors.

CONDENSATION DE BULLES DE VAPEUR UN LIQUIDE

Résumé—On étudie la condensation d'une bulle de vapeur à symétrie sphérique dans un liquide sous-refroidi donné, à partir de la première loi de la thermodynamique pour une masse variable. Les résultats théoriques sont comparés aux résultats expérimentaux disponibles obtenus par différents auteurs.

KONDENSATION VON DAMPFBLASEN IN FLÜSSIGKEITEN

Zusammenfassung—Auf Grund des ersten Satzes der Thermodynamik variabler Massen wird die Kondensation einer kugelsymmetrischen Dampfblase in einer ruhenden unterkühlten Flüssigkeit untersucht. Die theoretischen Ergebnisse werden mit den vorhandenen experimentellen Daten verschiedener Autoren verglichen.